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Cost Benefit Analysis Of Two Duplicate Units Parallel System With Correlation In Failure And Repair

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Abstract

A large number of authors have analysed the models of two-unit parallel/standby redundant system with various sets of assumptions. They assumed that a failed unit is always reparable and works as good as new. In practice we come across situations when we have duplicate units which are reparable only a finite number of times and then it is replaced by new ones. Also, on repair the duplicate unit might not be able to work as good as new but acts with less efficiency in quasi-normal mode. Keeping this fact in view, we in the present paper we analyse a two unit parallel systems model in which one units in considered as duplicate unit and it is reparable only once before its replacement with new ones

System comprises of two non-identical units. One unit is original and other is duplicate unit. Initially both the units work in parallel configuration. The original unit is reparable any number of times whereas the duplicate unit is reparable only once and after repair it does not work as good as new. The original unit becomes as good as new after each repair. The original unit has two modes – normal (N) and total failure (F) whereas duplicate unit has three modes – normal (N), quasi-normal (QN) and total failure (F). After each repair a duplicate unit enters into QN- mode. When duplicate unit fails second time i.e. enters from QN- mode to F-mode then it is nor reparable. In this situation the failed duplicate unit is replaced by new ones. A single repairman is always available with the system to repair a failed original and duplicate unit and to replace the failed duplicate unit. The duplicate unit in QN-mode works with reduced (less) efficiency as compared to its N-mode. The service discipline is first come first served (FCFS). The failure and repair/replacement times of each unit are assumed to be correlated random variables having their joint distributions as bivariate exponential (B.V.E.) . The p.d.f. of the B.V.E. is

$$f(x, y) = f(x, y) = \propto \beta(1-r)e^{-\propto x-\beta y}\, I_0\left(2\sqrt{\alpha\beta rxy}\right) \qquad x, y, \propto, \beta > 0 \;\; ; \;\; 0 \leq r < 1$$

Where, $I_0(Z) = \sum_{K=0}^{\infty}\frac{(z/2)^{2K}}{K!^2}$ is the modified Bessel function of type-I and order zero.

Notations And States Of The System: We define the following symbols for generating the various states of the system –

N_{10}, N_{20}	:	Original and duplicate unit in N (normal) mode and operative.
QN_{20}	:	Second unit in QN (quasi-normal) mode and operative with less efficiency.
F_{1r}, F_{1wr}	:	Original unit in F-mode and under repair/waiting for repair.
F_{2r}, F_{2wr}	:	Second unit in F-mode and under repair/waiting for repair.
F_{2rp}, F_{2wrp}	:	Second unit in F-mode and under replacement/waiting for replacement.

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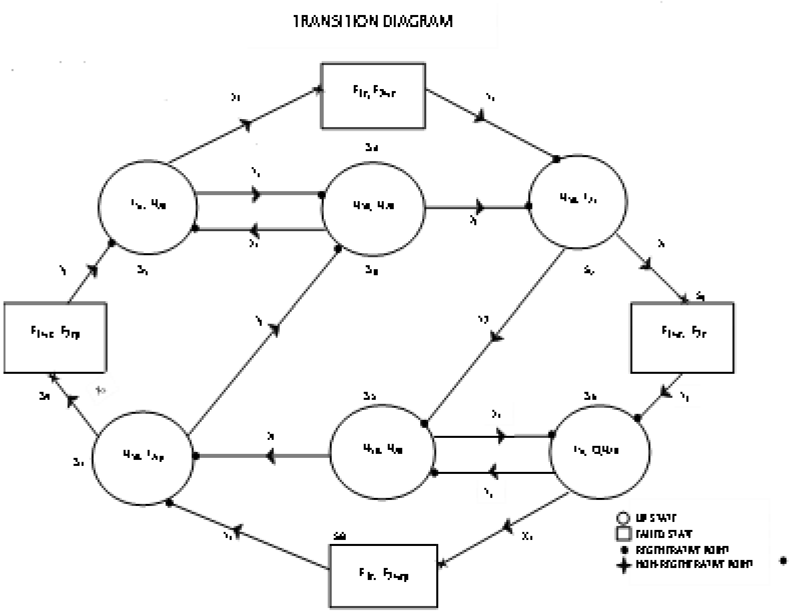
Considering the above symbols in view of assumptions stated , we have the following states of the system :

Up-states : $S_0 \equiv (N_{10}, N_{20})$, $S_1 \equiv (F_{1r}, N_{20})$, $S_2 \equiv (N_{10}, F_{2r})$, $S_5 \equiv (N_{10}, QN_{20})$, $S_6 \equiv (F_{1r}, QN_{20})$, $S_7 \equiv (N_{10}, F_{2rp})$; **Failed-states :** $S_3 \equiv (F_{1r}, F_{2wr})$, $S_4 \equiv (F_{1wr}, F_{2r})$, $S_8 \equiv (F_{1r}, F_{2wr})$, $S_9 \equiv (F_{1wr}, F_{p2r})$

The transition diagram of the system model along with failure/repair/replacement times is shown in following figure :



The other notations used in this paper are defined as follows:

X_1, X_2 : Random variables representing the failure times of original unit and duplicate unit when it is as good as new.

X_3 : Random variable representing the failure time of duplicate unit from the quasi-normal mode

Y_1, Y_2 : Random variable representing the repair times of original unit and duplicate unit.

Y_3 : Random variable representing the replacement time of duplicate unit.

$f_i(x, y)$: Join p.d.f. of (X_i, Y_i) ; i = 1, 2, $= \alpha_i \beta_i (1 - r_i) e^{-\alpha_i x - \beta_i y} I_0\left(2\sqrt{\alpha_i \beta_i r_i x y}\right)$ x, y, $\alpha_i, \beta_i > 0$; $0 \le r_i < 1$ where, $I_0\left(2\sqrt{\alpha_i \beta_i r_i x y}\right) = \sum_{j=0}^{\infty} \frac{(\alpha_i \beta_i r_i x y)^j}{(j\,!)^2}$

$g_i(\,.)$: Marginal p.d.f. of $X_i = \alpha_i (1 - r_i)\, e^{-\alpha_i (1 - r_i) x}$

$h_i(\,.)$: Marginal p.d.f. of $Y_i = \beta_i (1 - r_i)\, e^{-\beta_i (1 - r_i) y}$

$k_i(y \mid x)$: Conditional p.d.f. of Y_i given $x_i = x$ = $\beta_i e^{-\beta_i y - \alpha_i r_i x} I_0\left(2\sqrt{\alpha_i \beta_i r_i x y}\right)$

Transition Probabilities: The steady-state transition probabilities are obtained –

$p_{01} = \frac{\alpha_1(1-r_1)}{\alpha_1(1-r_1)+\alpha_2(1-r_2)}$ $\quad p_{02} = \frac{\alpha_2(1-r_2)}{\alpha_1(1-r_1)+\alpha_2(1-r_2)}$ $\quad p_{56} = \frac{\alpha_1(1-r_1)}{\alpha_1(1-r_1)+\alpha_3(1-r_3)}$

$p_{57} = \frac{\alpha_3(1-r_3)}{\alpha_1(1-r_1)+\alpha_3(1-r_3)}$ $\quad p_{10} = \frac{\beta_1'(1-r_1)}{(1-r_1\beta_1')}$ $\quad p_{13} = 1 - \frac{\beta_1'(1-r_1)}{(1-r_1\beta_1')}$

$p_{25} = \frac{\beta_2'(1-r_2)}{(1-r_2\beta_2')}$ $\quad p_{24} = 1 - \frac{\beta_2'(1-r_2)}{(1-r_2\beta_2')}$ $\quad p_{65} = \frac{\beta_1''(1-r_1)}{(1-r_1\beta_1'')}$

$p_{68} = 1 - \frac{\beta_1''(1-r_1)}{(1-r_1\beta_1'')}$ $\quad p_{70} = = \frac{\beta_3'(1-r_3)}{(1-r_3\beta_3')}$ $\quad p_{79} = 1 - \frac{\beta_3'(1-r_3)}{(1-r_3\beta_3')}$

Where $\beta_1' = \frac{\beta_1}{\beta_1+\alpha_2(1-r_2)}$; $\beta_2' = \frac{\beta_2}{\beta_2+\alpha_1(1-r_1)}$; $\beta_1'' = \frac{\beta_1}{\beta_1+\alpha_3(1-r_3)}$; $\beta_3' = \frac{\beta_3}{\beta_3+\alpha_1(1-r_1)}$

Now we observe the following relations –

$p_{01} + p_{02} = 1$; $p_{10} + p_{13} = p_{10} + p_{12}^{(3)} = 1$; $p_{25} + p_{24} = p_{25} + p_{26}^{(4)} = 1$

$p_{56} + p_{57} = 1$; $p_{65} + p_{68} = p_{65} + p_{67}^{(8)} = 1$; $p_{70} + p_{79} = p_{70} + p_{71}^{(9)} = 1$

The mean sojourn times in various states are as follows –

$$\Psi_0 = \frac{1}{\alpha_1(1-r_1)+\alpha_2(1-r_2)} \quad ; \quad \Psi_1 = \frac{1}{\alpha_2(1-r_2)}\left[1-\frac{\beta_1'(1-r_1)}{(1-r_1\beta_1')}\right]$$

$$\Psi_2 = \frac{1}{\alpha_1(1-r_1)}\left[1-\frac{\beta_2'(1-r_2)}{(1-r_2\beta_2')}\right] \quad ; \quad \Psi_5 = \frac{1}{\alpha_1(1-r_1)+\alpha_3(1-r_3)}$$

$$\Psi_6 = \frac{1}{\alpha_3(1-r_3)}\left[1-\frac{\beta_1''(1-r_1)}{(1-r_1\beta_2'')}\right] \quad ; \quad \Psi_7 = \frac{1}{\alpha_1(1-r_1)}\left[1-\frac{\beta_3'(1-r_3)}{(1-r_3\beta_3')}\right]$$

Analysis Of Reliability And MTSF: Let the random variable T_i be the time to system failure (TSF) when the system starts its operation from state $S_i \in E$. Then the reliability of the system is given by –$R_i(t) = P(T_i > t)$ To determine $R_i(t)$, we regard the failed states S_3, S_4, S_8 and S_9 of the system as absorbing. By elementary probabilistic arguments, we have the following system of integral equations –

$R_0(t) = Z_0(t) + q_{01}(t) © R_1(t) + q_{02}(t) © R_2(t)$

$R_1(t) = Z_1(t) + q_{10}(t) © R_0(t)$

$R_2(t) = Z_2(t) + q_{25}(t) © R_5(t)$

$R_5(t) = Z_5(t) + q_{56}(t) © R_6(t) + q_{57}(t) © R_7(t)$

$R_6(t) = Z_6(t) + q_{56}(t) © R_5(t)$

$R_7(t) = Z_7(t) + q_{70}(t) © R_0(t)$ (1-6)

$Z_0 = e^{-\{\alpha_1(1-r_1)+\alpha_2(1-r_2)\}\,t}$; $Z_1(t) = e^{-\alpha_2(1-r_2)\,t}\int\left\{\int_t^\infty \beta_1 e^{-(\beta_1 u+\alpha_1 r_1 x)} I_0\left(2\sqrt{\alpha_1\beta_1 r_1 xu}\right) du\right\}\times g_1(x)dx$

$Z_2(t) = e^{-\alpha_1(1-r_1)\,t}\int\left\{\int_t^\infty \beta_2 e^{-(\beta_2 u+\alpha_2 r_2 x)} I_0\left(2\sqrt{\alpha_2\beta_2 r_2 xu}\right) du\right\}\times g_2(x)\,dx$; $Z_5 = e^{-\{\alpha_1(1-r_1)+\alpha_3(1-r_3)\}\,t}$

$Z_6(t) = e^{-\alpha_3(1-r_3)\,t}\int\left\{\int_t^\infty \beta_1 e^{-(\beta_1 u+\alpha_1 r_1 x)} I_0\left(2\sqrt{\alpha_1\beta_1 r_1 xu}\right) du\right\}\times g_1(x)dx$ $Z_7(t) =$

$e^{-\alpha_1(1-r_1)\,t}\int\left\{\int_t^\infty \beta_3 e^{-(\beta_3 u+\alpha_3 r_3 x)} I_0\left(2\sqrt{\alpha_3\beta_3 r_3 xu}\right) du\right\}\times g_3(x)\,dx$

As an illustration, $R_0(t)$ is the sum of the following three mutually exclusive contingencies – System remains up in state S_0 without making any transition to any other state up to time t. The probability of this contingency is

$e^{-\{\alpha_1(1-r_1)+\alpha_2(1-r_2)\}\,t} = Z_0(t)$, say

System first enters the state S_1 from S_0 during (u, u + du) u $\leq$ t and then starting from S_1, it remains up without any breakdown for time duration (t – u). The probability of this contingency is,

$\int_0^t q_{01}(u)\,du\,R_1(t-u) = q_{01}(t) © R_1(t)$

System first enters the state S_2 from S_0 during (u, u +du), u $\leq$ t and then starting from S_2, it remains up without any breakdown for time duration (t – u). The probability of this contingency is–

$\int_0^t q_{02}(u)\,du\,R_2(t-u) = q_{02}(t) © R_2(t)$

Taking Laplace Transforms (L.T.) of the relations (1-6), we can write the solution of resulting set of algebraic equations in the matrix form as follows –

$$\begin{bmatrix} R_0^* \\ R_1^* \\ R_2^* \\ R_5^* \\ R_6^* \\ R_7^* \end{bmatrix} = \begin{bmatrix} 1 & -q_{01}^* & -q_{02}^* & 0 & 0 & 0 \\ -q_{10}^* & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -q_{25}^* & 0 & 0 \\ 0 & 0 & 0 & 1 & -q_{56}^* & -q_{57}^* \\ 0 & 0 & 0 & -q_{65}^* & 1 & 0 \\ -q_{70}^* & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} Z_0^* \\ Z_1^* \\ Z_2^* \\ Z_5^* \\ Z_6^* \\ Z_7^* \end{bmatrix}$$

For brevity, we have omitted the argument's' from $q_{ij}^*(s)$, $Z_i^*(s)$ and $R_i^*(s)$.. Computing the above matrix equation for $R_0^*(s)$, we get – $R_0^* = \frac{N_1(s)}{D_1(s)}$

Where, $N_1(s) = \left(1 - q_{56}^* q_{65}^*\right)\left(Z_0^* + q_{01}^* Z_1^* + q_{02}^* Z_2^*\right) + q_{02}^* q_{25}^* (Z_5^* + q_{56}^* Z_6^* + q_{57}^* Z_7^*)$ and,

$D_1(s) = (1 - q_{56}^* q_{65}^*)(1 - q_{01}^* q_{10}^*) - q_{02}^* q_{25}^* q_{57}^* q_{70}^*$

Taking the Inverse Laplace Transform of (8), one can get the reliability of the system when initially system starts from S_0. The mean time to system failure (MTSF) can be obtained using the well known formula –

$$E(T_0) = \int R_0(t)d = \lim_{s\to 0} R_0^*(s) = \frac{N_1(0)}{D_1(0)} \quad (9)$$

To determine $N_1(0)$ and $D_1(0)$, we first obtain $Z_i^*(0)$, using the result, $\lim_{s\to 0} Z_i^*(s) = \int Z_i(t)dt$

Therefore $Z_0^*(0) = \Psi_0$, $Z_1^*(0) = \Psi_1$, $Z_2^*(0) = \Psi_2$, $Z_5^*(0) = \Psi_5$,

$Z_6^*(0) = \Psi_6$, $Z_7^*(0) = \Psi_7$ Thus, using $q_{ij}^*(0) = q_{ij}$ and the above results we get,

$N_1(0) = (1 - p_{56}p_{65})(\Psi_0 + p_{01}\Psi_1 + p_{02}\Psi_2) + p_{02}p_{25}(\Psi_5 + p_{56}\Psi_6 + p_{57}\Psi_7)$ and,

$D_1(0) = (1 - p_{56}p_{65})(1 - p_{01}p_{10}) - p_{02}p_{25}p_{57}p_{70}$

Availability Analysis: It is obvious that out of six up states S_0, S_1, S_2, S_5, S_6, S_7, the system works with full efficiency in five states namely S_0, S_1, S_2, S_5 and S_7 whereas in state S_6 system works with reduced efficiency. So, accordingly let us define $A_i^1(t)$ and $A_i^2(t)$ as the probabilities that the system is up (operative) at epoch t with full efficiency and reduced efficiency respectively when system initially starts from state $S_i \in E$. By simple probabilistic laws, we observe that $A_0^1(t)$ is the sum of the following mutually exclusive contingencies –

$$A_0^1(t) = Z_0(t) + q_{01}(t) © A_1^1(t) + q_{02}(t) © A_2^1(t)$$

$$A_0^1(t) = Z_1(t) + q_{10}(t) © A_0^1(t) + q_{12}^{(3)}(t) © A_2^1(t)$$

$$A_2^1(t) = Z_2(t) + q_{25}(t) © A_5^1(t) + q_{26}^{(4)}(t) © A_6^1(t)$$

$$A_5^1(t) = Z_5(t) + q_{57}(t) © A_7^1(t) + q_{56}(t) © A_6^1(t)$$

$$A_6^1(t) = q_{65}(t) © A_5^1(t) + q_{67}^{(8)}(t) © A_7^1(t)$$

$$A_7^1(t) = Z_7(t) + q_{70}(t) © A_0^1(t) + q_{71}^{(9)}(t) © A_1^1(t)$$

Taking Laplace Transform (L.T.) of the above relations and writing the solution of the resulting set of algebraic equations as $A_0^{1*}(s) = \frac{N_s(s)}{D_2(s)}$ $N_2(s) = \Big[1 - q_{56}^* q_{65}^* - q_{71}^{(9)*} q_{12}^{(3)*} \Big\{ q_{25}^* (q_{56}^* q_{67}^{(8)*} + q_{57}^*) + q_{26}^{(4)*} (q_{67}^{(8)*} + q_{65}^* q_{57}^*) \Big\} \Big] Z_0^* + \Big[q_{01}^* \big(1 - q_{56}^* q_{65}^*\big) + q_{71}^{(9)*} q_{02}^* \Big\{ q_{25}^* \big(q_{56}^* q_{67}^{(8)*} + q_{57}^* \big) + q_{26}^{(4)*} \big(q_{67}^{(8)*} + q_{65}^* q_{57}^* \big) \Big\} \Big] Z_1^* + \big(1 - q_{56}^* q_{65}^*\big) \big(q_{01}^* q_{12}^{(3)*} + q_{02}^* \big) Z_2^* + \big(q_{25}^* + q_{26}^{(4)*} q_{65}^* \big) \big(q_{01}^* q_{12}^{(3)*} + q_{02}^* \big) Z_5^* + \big(q_{01}^* q_{12}^{(3)*} + q_{02}^* \big) \times \Big\{ q_{25}^* \big(q_{56}^* q_{67}^{(8)*} + q_{57}^* \big) + q_{26}^{(4)*} \big(q_{67}^{(8)*} + q_{65}^* q_{57}^* \big) \Big\} Z_7^*$ and

$$D_2(s) = (1 - q_{56}^*)(1 - q_{01}^* q_{10}^*) - \big((q_{71}^{(9)*} q_{12}^{(3)*} + q_{10}^* q_{02}^* q_{71}^{(9)*} + q_{01}^* q_{12}^{(3)*} q_{70}^* + q_{02}^* q_{70}^*) \Big\{ q_{25}^* (q_{56}^* q_{67}^{(8)*} + q_{57}^*) + q_{26}^{(4)*} (q_{67}^{(8)*} + q_{65}^* q_{57}^*) \Big\}$$

Similarly, the recurrence relations among point wise availabilities $A_j^2(t)$ i.e. when the system is up at epoch t with reduced efficiency, can be obtained as follows

$A_0^2(t) = q_{01}(t) © A_0^2(t) + q_{02}(t) © A_2^2(t)$

$A_1^2(t) = q_{10}(t) © A_0^2(t) + q_{12}^{(3)}(t) © A_2^2(t)$

$A_2^2(t) = q_{25}(t) © A_5^2(t) + q_{26}^{(4)}(t) © A_6^2(t)$

$A_5^2(t) = q_{57}(t) © A_7^2(t) + q_{56}(t) © A_6^2(t)$

$A_6^2(t) = Z_6(t) + Z_{65}(t) © A_5^2(t) + q_{67}^{(8)}(t) © A_7^2(t)$

$A_7^2(t) = q_{70}(t) © A_0^2(t) + q_{71}^{(9)}(t) © A_1^2(t)$

Taking L.T. of above relations and solving for $A_0^{2*}(s)$, we get $A_0^{2*}(s) = N_3(s)/D_2(s)$

Where, $N_3(s) = (q_{01}^* q_{12}^{(3)*} + q_{02}^*)(q_{25} q_{56} + q_{26}^{(4)*}) Z_6^*$

Busy Period Analysis: Let $B_i^1(t)$, $B_i^2(t)$ and $B_i^3(t)$ be the respective probabilities that the repairman is busy in the repair of original unit, duplicate unit and in replacement of duplicate unit at time t when system initially starts from regenerative state S_i. Using simple probabilistic arguments we have the system of integral equations for $B_i^1(t)$ as follows –

$B_0^1(t) = q_{01}(t) © B_1^1(t) + q_{02}(t) © B_2^1(t)$

$B_1^1(t) = \int \overline{K_1}(t\,|x)\, g_1(x)dx + q_{10}(t) © B_0^1(t) + q_{12}^{(3)}(t) © B_2^1(t)$

$B_2^1(t) = q_{25}(t) © B_5^1(t) + q_{26}^{(4)}(t) © B_6^1(t)$

$B_5^1(t) = q_{57}(t) © B_7^1(t) + q_{56}(t) © B_6^1(t)$

$B_6^1(t) = \int \overline{K_1}(t\,|x)\, g_1(x)dx + q_{65}(t) © B_5^1(t) + q_{67}^{(8)}(t) © B_7^1(t)$

$B_7^1(t) = q_{70}(t) © B_0^1(t) + q_{71}^{(9)}(t) © B_1^1(t)$

Taking L.T. of above algebraic equations for $B_1^1(t)$, we get , $B_0^{1*}(s) = \frac{N_4(s)}{D_2(s)}$ $N_4(s) = q_{01}^*(1 - q_{56}^* q_{65}^*) - q_{71}^{(9)*} q_{02}^* \big(q_{25}^*(q_{56}^* q_{67}^{(8)*} + q_{57}^*) + q_{26}^{(4)*}(q_{67}^{(8)*} + q_{65}^* q_{57}^*)\big) + (q_{01}^* q_{12}^{(3)*} + q_{02}^*)(q_{25}^* q_{56}^* + q_{26}^*)\big]\, W_1^*(s)$ and

$W_1^*(s) = L.T.[\int \overline{K_1}(t\,|x)\, g_1(x)\, dx]$ and $D_2(s)$ is same as given earlier.

Similarly, the recurrence relations in $B_i^2(t)$ may be developed as follows –

$B_0^2(t) = q_{01}(t) © B_1^2(t) + q_{02}(t) © B_2^2(t)$

$B_0^2(t) = q_{10}(t) © B_1^2(t) + q_{12}^{(3)}(t) © B_2^2(t)$

$B_2^2(t) = \int \overline{K_2}(t\,|x)\, g_2(x)dx + q_{25}(t) © B_5^2(t) + q_{26}^{(4)}(t) © B_6^2(t)$

$B_5^2(t) = q_{57}(t) © B_7^2(t) + q_{56}(t) © B_6^2(t)$

$B_6^2(t) = q_{65}(t) © B_5^2(t) + q_{67}^{(8)}(t) © B_7^2(t)$

$B_7^2(t) = q_{70}(t) © B_0^2(t) + q_{71}^{(9)}(t) © B_1^2(t)$ (8-13)

Taking L.T. of above relations and solving the resulting set of equations, we get, $B_0^{2*}(s) = \frac{N_5(s)}{D_2(s)}$ where

$N_5(s) = (1 - q_{56}^* q_{65}^*)(q_{01}^* q_{12}^{(3)*} + q_{02}^*)W_2^*(s)$ and $W_2^*(s) = L.T.[\int \overline{K_2}(t\,|x)\, g_2(x)\, dx]$

Finally, the recurrence relation in$B_1^3(t)$ are as follow –

$B_0^3(t) = q_{01}(t) © B_1^3(t) + q_{02}(t) © B_2^3(t)$

$B_0^3(t) = q_{10}(t) © B_0^3(t) + q_{12}^{(3)}(t) © B_2^3(t)$

$B_2^3(t) = q_{25}(t) © B_5^3(t) + q_{26}^{(4)}(t) © B_6^3(t)$

$B_5^3(t) = q_{57}(t) © B_7^3(t) + q_{56}(t) © B_6^3(t)$

$B_6^3(t) = q_{65}(t) © B_5^3(t) + q_{67}^{(8)}(t) © B_7^3(t)$

$B_7^3(t) = \int \overline{K_3}(t\,|x)\, g_3(x)dx + q_{70}(t) © B_0^3(t) + q_{71}^{(9)}(t) © B_1^3(t)$

The value of $B_0^{3*}(s)$ after taking L.T. of above relations and solving them we get $B_0^{3*}(s) = \frac{N_6(s)}{D_2(s)}$:

Where, $N_6(s) = (q_{01}^* q_{12}^{(3)*} + q_{02}^*)[q_{25}^*(q_{56}^* q_{67}^{(8)*} + q_{57}^*) + q_{26}^{(4)*}(q_{67}^{(8)*} + q_{65}^* q_{57}^*)]W_3^*(s)$ and

$W_3^*(s) = L.T.[\int \overline{K_3}(t\,|x)\, g_3(x)\, dx]$

Now to obtain the steady-state probabilitiesB_0^1, B_0^2 and B_0^3 that the repairman is busy in the repair of original unit, duplicate unit and in replacement of duplicate unit, we use the results –

$N_4(0) = [(1 - p_{56}p_{65})(1 - p_{70}p_{02}) + (1 - p_{01}p_{10})(1 - p_{25}p_{57})]\frac{1}{\beta_1(1-r_1)}$

$N_5(0) = (1 - p_{56}p_{65})(1 - p_{01}p_{10})\frac{1}{\beta_2(1-r_2)}$; $N_6(0) = (1 - p_{01}p_{10})(1 - p_{56}p_{65})\frac{1}{\beta_3(1-r_3)}$

And $D_2'(0)$ is same as defined earlier

Profit Function Analysis: We now obtain the cost function of the system considering the mean up time of the system in normal and quasi-normal modes during (0, t) and expected busy of repairman in the repair of original unit, duplicate unit and in replacement of duplicate unit during (0, t). The net expected cost/profit (gain) incurred during (o, t) is given by,

P(t) = Expected total revenue in (0, t) – Expected total expenditure during (0, t)

$$= K_0\mu_{up}^1(t) + K_1\mu_{up}^2(t) - K_2\mu_b^1(t) - K_3\mu_b^2(t) - K_4\mu_b^3(t) - K_5N_0\ (t) \qquad (1)$$

Where K_0 and K_1 are the revenues per-unit up time by the system in normal (N) mode and quasi-normal (QN) mode respectively; K_2, K_3 and K_4 are the amounts paid to the repairman per-unit of time when he is busy in repairing the original unit, duplicate unit and in replacement of duplicate unit respectively; K_5 is the cost of a duplicate unit, Also,

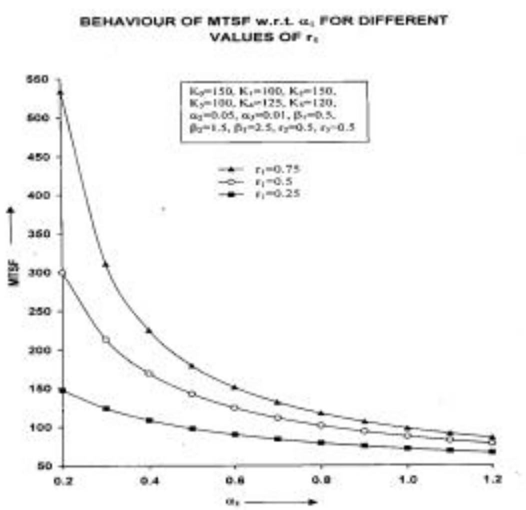
$\mu_{up}^1(t) = \int_0^t A_0^1(u)\ du$ s.t $\mu_{up}^{1*}(s) = A_0^{1*}(s)/s$; $\mu_{up}^2(t) = \int_0^t A_0^2(u)\ du$ s.t. $\mu_{up}^{2*}(s) = A_0^{2*}(s)/s$

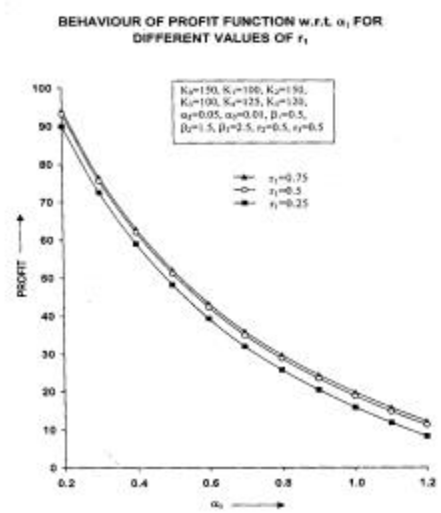
$\mu_b^1(t) = \int_0^t B_0^1(u)\ du$ s.t. $\mu_b^{1*}(s) = B_0^{1*}(s)/s$ $\mu_b^2(t) = \int_0^t B_0^2(u)\ du$ s.t. $\mu_b^{2*}(s) = B_0^{2*}(s)/s$

$\mu_b^3(t) = \int_0^t B_0^3(u)\ du$ s.t. $\mu_b^{3*}(s) = B_0^{3*}(s)/s$

Now the expected total profit per-unit time in steady-state is given by,

$$P = \lim_{t\to\infty} P\,(t)/t = \lim_{s\to\infty} s^2\,P^*(s) = K_0A_0^1 + K_1A_0^2 - K_2B_0^1 - K_3B_0^2 - K_4B_0^3 - K_5N_0$$





Study Of System Behaviour Through Graphs:

For this study, we plot curves for MTSF and profit function with respect to a_1 (failure parameter of original unit) for three different values of r_1 (0.25, 0.50, 0.75), the correlation coefficient between failure and repair times of original unit. The other parameters are kept fixed as $a_2 = 0.05$, $a_3 = 0.01$, $\beta_1 = 0.5$, $\beta_2 = 1.5$, $\beta_3 = 2.5$, $r_2 = r_3 = 0.5$, $K_0 = 150$, $K_1 = 100$, K_2, 150, $K_3 = 100$, $K_4 = 125$, $K_5 = 120$. From Fig. 2.2, it is clear that the MTSF decreases uniformly as a_1 increases where as it increases with the increase in correlation coefficient r_1. The similar trends are observed in figure for the case of profit in respect of a_1 and r_1. Thus we conclude that the higher correlation between failure and repair times provides the better system performances.

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