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Hydromagnetic Flow Of Rivlin-Ericksen Fluid Through a Porous Medium Between Two Inclined Parallel Plats

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Abstract

Purpose of this paper is to study the hydromagnetic flow of Rivlin-Ericksenfluid through porous medium between two inclined parallel plates. In this paper, we have considered the flow of two immiscible conducting visco-elastic fluids between two parallel plates in presence of transverse uniform magnetic field. The pressure gradient of the flow are either transient or periodic in nature.

Introduction

The aim of present investigation is to study the hydromagnetic flow of Rivilin-Ericksen fluid through a porous medium between two inclined parallel plates. The stability of a horizontal layer of viscoelastic fluid heated from below has been investigated by Vest and Arpaci [1]. Rechardson and Taylor [3] investigated the oscillatory flow of fluid through a long circular tube under the influence of periodic pressure gradient. Bhatia and Steiner [11] have considered the effect of a uniform rotation on the thermal instability of a Maxwell fluid and have found that rotation has a destabilizing effect in contrast to the stabilizing effect on Newtonian fluid. The flow of a viscous incompressible fluid past a Semi-infinite plate started impulsively from rest, by analytical method where as it was studied by finite difference method by Hall [8] and by integral method by Tani and Yu [5]. Bhatia and Steiner [12] have also studied the problem of thermal instability of a viscoelastic fluid in hydromagnetics and have found that the magnetic field has the stabilizing influence on Maxwell fluid just as in the case of Newtonian fluid. The thermal instability of a viscoelastic (oldoydian) fluid has been considered in the presence of magnetic field by Sharma [14] and in the presence of suspended particles by Sharma and Sharma [15].

Here an attempt is made to solve the problem given by Das [2] for two immiscible conducting viscoelastic fluid between parallel plates.

In this chapter, we have considered the flow of two immiscible conducting visco-elastic fluids between two parallel plates in presence of transverse uniform magnetic field. The pressure gradient of the flow are either transient or periodic in nature.

Formulation of The Problem

Here we have assumed the following notations :

t	=	Time variable
B_0	=	Magnetic induction
u	=	Velocity of flow
λ	=	Kinematic coefficient of visco-elasticity

In the present analysis, we have assumed that unlimited mass of conducting visco-elastic fluid is separated by two parallel plates of depth 2a. Purterbation of the field has been ignored. In this chapter, the fluid was initially at rest and both the fluids separated by the plates were set in motion under the action of time varying pressure gradient. Origin lies on lower plates which is fixed.

The flow of fluids is along x-axis and perpendicular to y-axis.

In this regard, the equation of motion can be written in the form :

(2.1) $$\frac{\partial u}{\partial t}=\left(V+\lambda\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2 u}{\rho}-\frac{1\partial\rho}{\rho\partial x}$$

(Wherein λ is kinematic coefficient of visco-elasticity)

Equation of continuity is given by

(2.2) $$\frac{\partial u}{\partial x}=0$$

Solution of The Problem

We have considered following two cases :

Case I : In the first case, we have considered that upper plate moves with transient velocity $\omega' e^{-t}$ and the transient pressure gradient πe^{-t} is applied to the two fluids.

(3.1) i.e. $u = \omega' e^{-t}$

and

(3.2) $$\frac{\partial p}{\partial x}=\pi e^{-t}$$

From equation (3.1), (3.2) and (2.1), we obtain

(3.3) $$\frac{\partial^2\omega'}{\partial y^2}-\frac{\sigma B_0^2-\rho}{\rho(V-\lambda)}\omega'=\frac{\pi}{\rho(V-\lambda)}$$

equation (3.3) can be written as

(3.4) $$\frac{\partial^2\omega'}{\partial y^2}-A^2\omega'=B^2$$

where

(3.5) $$\frac{\sigma B_0^2-\rho}{\rho(V-\lambda)}-A^2$$

and

(3.6) $$\frac{\pi}{\rho(V-\lambda)}=B^2$$

The boundary conditions are as follows :

(3.7)	$\omega_1 = 0$	when	$y = -a$	
	$\omega_2 = u_1$	when	$y = 0$	$t \geq 0$
(3.8)	$\omega_1 = u_1$	when	$y = 0$	
	$\omega_2 = u_2$	when	$y = a$	$t \geq 0$

In this regard, we shall consider following two conditions :

Condition (A)

Let us consider that the first fluid moves under the boundary conditions (3.7), then the solution of the equation (3.4) becomes

(3.9) $$\omega'=\left(u_1+\frac{B^2}{A^2}\right)\cos hAy+\left\{\left(u_1+\frac{B^2}{A^2}\right)\cos hAa-\frac{B^2}{A^2}\operatorname{cosec} hAa\right\}$$

$$\sin hAy-\frac{B^2}{A^2}$$

Subsituting equation (3.9) into equation (3.1), we obtain

(3.10) $u = [(u_1 + v')\, \cosh Ay + \{(u_1 + v')\, \cot hAa – v'\, \operatorname{cosec} hAa\}$

sinhAy -v'] e^{-t}

Wherein $v' = \frac{B^2}{A^2}$

Remark 3.1

It is to be noted that if we take A is very small, then

$\cosh Ay \cong 1$ and $\sin hAy \cong Ay$

then the relation (3.10) reduces in the form

(3.11) $$u = u_1\left(1 + \frac{y}{a}\right)e^{-t}$$

This shows that the velocity is transient in character and approaches to zero after a long time from the start of the motion.

Condition (B)

Let us consider that second fluid moves under the boundary conditions (3.8), the solution of equation (3.4) becomes –

(3.12) $$\omega' = \left(u_1 + \frac{B^2}{A^2}\right)\cos hAy +$$

$$\left\{\left(u_2 + \frac{B^2}{A^2}\right)\operatorname{cosech}Aa - \left(u_1 + \frac{B^2}{A^2}\right)\cot hAa\right\}\sin hAy - \frac{B^2}{A^2}$$

(3.13) u = [(u$_1$ + v') coshAy + {u$_2$ + v') cosec hAa – (u$_1$ + v') cot hAa]

sinhAy – v'] e^{-t}

wherein $v' = \frac{B^2}{A^2}$

If a is much smaller, then we take

(3.14) $\cosh Ay \cong 1$ and $\sin hAy \cong Ay$

The equation (3.13) reduces in the form

(3.15) $$u=\left[\left(1-\frac{y}{h}\right)+u_2\frac{y}{h}\right]e^{-t}$$

Case II : In this case, we have assumed that upper plate is moving parallel to itself with periodic velocity $\omega' e^{it}$ and periodic pressure gradient πe^{it} is applied to the two fluids.

(3.16) i.e. $u = \omega' e^{it}$

and

(3.17) $$\frac{\partial \rho}{\partial x}=\pi e^{it}$$

By virtue of equation (3.16) and equation (3.17) and (2.1), we get -

(3.18) $$\frac{\partial^2 \omega'}{\partial y^2}-\left(\frac{\left(\sigma B_0^2+i\rho\right)}{\rho(V+i\lambda)}\right)\omega'=\frac{\pi}{\rho(V+i\lambda)}$$

If we choose

(3.19) $$\frac{\sigma B_0^2+i\rho}{\rho(V+i\lambda)}=C^2$$

and

(3.20) $$\frac{\pi}{\rho(V+i\lambda)}=D^2$$

then the equation (3.18) reduces in the form

(3.21) $$\frac{d^2\omega'}{dy^2}-C^2\omega'=D^2$$

The boundary conditions are as follows :

(3.22) $\omega_1 = 0$ when $y = -a$

$\omega_2 = C_0$ when $y = 0$ $t \geq 0$

(3.23) $\omega_1 = C_0$ when $y = 0$

$\omega_2 = C_1$ when $y = a$ $t \geq 0$

Now again following two conditions may by arise :

Condition [A]

Let us consider that the first fluid moves under the boundary condition (3.22), the solution of equation (3.21) takes the form

(3.24) $\omega' = (V_0 + \mu') \text{coshcy} + \{(V_0 + \mu') \cot hCa - \mu' \text{cosec } hCa]$

$\sinh Cy - \mu'$

wherein $\mu' = \frac{D^2}{C^2}$

Inserting the equation (3.24) into the equation (3.16) and (3.17), we obtain

(3.25) $u = [(v_0 + \mu') \cosh Cy + \{(v_0 + \mu') \cot hCa - \mu' \text{cosec } Ca]$

$\sinh Cy - \mu'] e^{it}$

If we take C is much smaller, then

(3.26) $\cosh Cy \cong 1$ and $\sin hCy \cong Cy$

In this regard, relation (3.25) takes the form

(3.27) $u = C_0 \left(1 + \frac{y}{t}\right) e^{it}$

Above expression indicates that the velocity u is a periodic function of time which depends on the depth of the fluids.

Condition [B]

In this case, we have considered the second fluid moves under the boundary condition (3.23), then the solution of equation (3.21) takes the form

(3.28) $\omega' = (v_0 + \mu') \cosh Cy + \{(v_1 + \mu') \operatorname{cosec} hCa - (v_0 + \mu') \cot hCa\}$

$\operatorname{Sin} hCy - \mu'$

Wherein $\mu' = \frac{D^2}{C^2}$

using (3.27) into the relation (3.16) and (3.17), we get

(3.29) $u = (v_0 + \mu') \cosh Cy + \{(v_1 + \mu') \operatorname{cosec} hCa - (v_0 + \mu') \cot hCa\}$

$\operatorname{Sin} hCy - \mu'] e^{it}$

using statement giving in equation (3.26), then the above equation becomes

(3.30) $$\boxed{u = \left[V_0 + (v_1 - v_0)\frac{y}{a} \right] \exp(it)}$$

Which shows that u is a periodic function of t and also depends on the depth of the fluids.

Remark 3.2

If $a \to \infty$, then relation (3.30) reduces in the form

$$\boxed{u = V_0 \exp(it)}$$

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