NJESR/May2020/Volume-2/Issue-5 **E-ISSN 2582-5836**
DOI-10.53571/NJESR2020.2.5.7-16

An Imprecise Inventory Model With Selling Price Dependent Demand For Time Dependent Decaying Items

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(Received: 20 April 2020/Revised: 10 May 2020/Accepted: 16 May 2020 Published: 26 May 2020)

Abstract
In this paper we developed an inventory model for decaying items in crisp and fuzzy environment. Inventory is continuously reviewed. Demand rate is taken as price sensitive. In this study shortages are allowed which is partially backlogged. In this model we consider single vendor and a single buyer for a single product. The production rate is finite and is greater than the sum of all the buyer demand.

Introduction

Analysis of an inventory control system is one of the outstanding subjects in operations research and industrial engineering. It is important for the decision-maker of inventory to identify all the deriving factors which produces effects on whole inventory planning. Acting as the driving force of the whole inventory system, demand is a key factor that should be taken into consideration while developing inventory control policy. Fuzzy set theory provides an alternative method for dealing with this kind of uncertainty from a broader perspective. Maximum physical goods undergo decay or deterioration over time. So decay or deterioration of physical goods in stock is a very realistic feature and inventory. Selling price plays an important role in inventory system.

Gupta and Vrat (1986) were amongst the first few researchers to deliberate the effects of stock dependent consumption rate on an EOQ model. In this study they established EOQ for two cases, one for an instantaneous replenishment and another for a finite rate of replenishment. Baker and Urban (1988) analyzed a continuous deterministic case of an inventory system in which the demand rate is a polynomial function of the inventory level. The algorithm using separable programming was employed to find the optimal solution. Mandal and Phaujdar (1989**)** wrote a note an inventory model with instantaneous stock replenishment and stock dependent consumption rate. Datta and Pal (1990) developed an inventory model with stock dependent demand until the stock level reached a particular point, after which the demand became constant. Giri et. al. (1996) extended Datta and Pal by relaxing their restriction of zero inventories at the end of order cycle and including deterioration effects. Rao et. al.(2004) developed an EOQ model with Weibull deterioration and stock dependent demand for an inventory with shortages. Optimal production and pricing policies to maximize the net present value of the total profit over a finite planning horizon was obtained. In (2001) Wee and Law studied an EOQ model with Weibull deterioration, price dependent demand considering the time value of money. Chen and Chen (2005) considered a decaying product with a price dependent and time varying demand. The decision model proposed solves optimally the production lot-size/scheduling problem taking into account the dynamic aspects of customer's demand as well as the restrictions of finite capacity in a plant. Roy and Chaudhuri (2006) studied a model with stock dependent demand under inflation and constant deterioration. Roy et. al. (2007) developed an inventory model for a seasonal product with deterioration which has a demand rate depending linearly upon displayed stock level.

There has been a lot of significant research on selling price dependent demand rate also, as is quite evident from the references cited below. Wee (1997) presented a replenishment policy for items with price dependent demand and a varying rate of deterioration. Wee and Law (1999) presented an optimization framework applying the DCF approach to an inventory with price dependent demand. Lately Dye (2007) developed a deterministic inventory model for deteriorating items with time dependent backlogging rate.

The model is developed with the following assumptions and notations.

Assumptions

The mathematical model is developed under the following assumptions.

1. There is no replacement or repair of deteriorated units.
2. Inventory is continuously reviewed.
3. Demand is price sensitive.

4. Shortages are allowed for buyers only, which is partially backlogged.
5. There are single vendor and a single buyer for a single product in this model.
6. The production rate is finite and is greater than the sum of all the buyer demand.
7. The deterioration rate is taken as $a+bt$.

Notations

p Purchase cost.
D Demand rate for vender $D=c-dP_v$ where c, d are the constants.
R Demand rate for buyer $R=c-dP_b$ where c, d are the constants.
KD The production rate per year where K > 1
T Time length of each cycle, where $T=T_1+T_2$.
T_1 The length of production runs.
T_2 The length of non production time.
$I_v(t)$ Inventory level for the vender at any time t.
$I_b(t)$ Inventory level for the buyer at any time t.
n Delivery times per period T for buyer.
I_{mv} The maximum inventory level for the vender.
I_{mi} The maximum inventory level for the buyer.
S_v The set up cost for each production cycle for vender.
S_b Ordering cost per order.
h_1, h_2 Holding cost per unit for the vender & buyer.
C_b, C_v Deterioration cost per unit for the vender and the buyer.
P_v Vender's retail price.
P_b Buyer's retail price.
O_b Opportunity cost per unit for the buyer.
Q Maximum ordered quantity for the buyer.

Mathematical Formulation

The Vender's Inventory Model

The total cycle time is divided into two periods, T_1 and T_2 . Where T_1 is the production period and T_2 is the non production period.

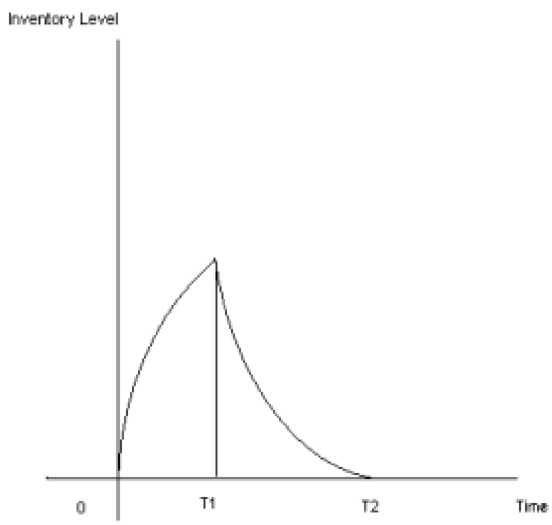

Fig - 1

The inventory system is represented by the following differential equations –

$$\frac{dI_v(t)}{dt} = -(a+bt)\,I_v(t) + D(K-1)\,, \quad 0 \le t \le T_1 \qquad \dots (1)$$

$$\frac{dI_v(t)}{dt} = -(a+bt)\,I_v(t) - D \qquad , \quad 0 \le t \le T_2 \qquad \dots (2)$$

With boundary conditions

$$I_v(0) = 0,\; I_v(T_2) = 0 \qquad \dots (3)$$

Solving equation (1) –

$$I_v(t)\,e^{(at+bt^2/2)} = D(K-1)(t + \frac{at^2}{2} + \frac{bt^3}{6}) + c_1 \qquad \dots (4)$$

Using boundary condition $I_v(0) = 0$, $c_1 = 0$

Put this value of c_1 in equation (4) –

$$I_v(t) = D(K-1)(t + \frac{at^2}{2} + \frac{bt^3}{6})e^{(-at-bt^2/2)}\,, \quad 0 \le t \le T_1 \qquad \dots (5)$$

Solving equation (2) –

$$I_{v_2}(t)\,e^{(at+bt^2/2)} = -D\,(t + \frac{at^2}{2} + \frac{bt^3}{6}) + c_2 \qquad \dots (6)$$

Using boundary condition $I_{v_2}(T_2) = 0$

$$I_{v_2}(t) = D\left\{\left(T_2 - t\right) + \frac{a}{2}\left(T_2^2 - t^2\right) + \frac{b}{6}\left(T_2^3 - t^3\right)\right\}e^{(-at-bt^2/2)}\,, \quad 0 \le t \le T_2 \qquad \dots (7)$$

From equation (7) –

$$I_{mv} = I_{v_2}(t)\,at\,t = 0$$

$$I_{mv} = D\left(T_2 + \frac{aT_2^2}{2} + \frac{bT_2^3}{6}\right) \qquad \dots (8)$$

By the boundary condition $I_{v_1}(T_1) = I_{v_2}(0)$, one can drive the following equation –

$$I_{v_1}(T_1) = D(K-1)\left(T_1 + \frac{aT_1^2}{2} + \frac{bT_1^3}{6}\right)e^{(-aT_1-bT_1^2/2)}$$

$$I_{v_2}(0) = D\left(T_2 + \frac{aT_2^2}{2} + \frac{bT_2^3}{6}\right)$$

$$(K-1)\left(T_1 + \frac{aT_1^2}{2} + \frac{bT_1^3}{6}\right)e^{(-aT_1-bT_1^2/2)} = \left(T_2 + \frac{aT_2^2}{2} + \frac{bT_2^3}{6}\right) \qquad \dots (9)$$

The Buyer's Inventory Model

At the beginning of each cycle I_{mi} units of item arrive at the inventory system. During the time interval $\left[0, t_1\right]$, the inventory level depletes due to combined effect of demand and deterioration. After it the shortage occurs to the end of the current order cycle. The whole process is repeated.

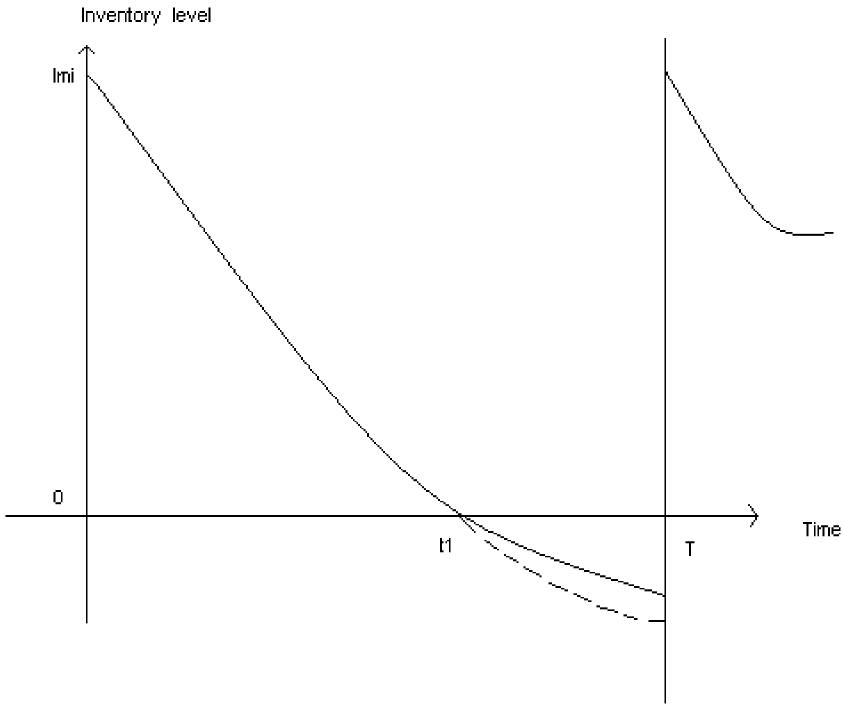

fig-2

The differential equations governing the transition of the system for the buyer is given by –

$$\frac{dI_b(t)}{dt}+(a+bt)I_b(t)=-R\ ,\quad 0\le t\le t_1 \qquad \dots (10)$$

$$\frac{dI_b(t)}{dt}=-RB\left(\frac{T}{n}-t\right)$$

$$\frac{dI_b(t)}{dt}=\frac{-R}{1+\delta\left(\frac{T}{n}-t\right)}\ ,\quad t_1\le t\le T/n \qquad \dots (11)$$

With boundary conditions –

$$I_b(t_1)=0$$

Solving equation (10) –

$$I_b(t)\,e^{\left(at+bt^2/2\right)}=-R\left(t+\frac{at^2}{2}+\frac{bt^3}{6}\right)+c_2 \qquad \dots (12)$$

$$I_b(t)=R\left\{(t_1-t)+\frac{a}{2}\left(t_1^2-t^2\right)+\frac{b}{6}\left(t_1^3-t^3\right)\right\}e^{\left(-at_1-bt_1^2/2\right)}\ ,\quad 0\le t\le t_1 \qquad \dots (13)$$

$$I_b(t)=\frac{R}{\delta}log\left\{1+\delta\left(\frac{T}{n}-t\right)\right\}+c_3 \qquad \dots (14)$$

Using boundary condition $I_b(t_1)=0$

$$I_b(t)=\frac{R}{\delta}\left[log\ \left\{1+\delta\left(\frac{T}{n}-t\right)\right\}-log\ \left\{1+\delta\left(\frac{T}{n}-t_1\right)\right\}\right],\quad t_1\le t\le T/n \qquad \dots (15)$$

At $t=T/n$ it will be the maximum backlogged demand so –

$$S = -I_b\left(\frac{T}{n}\right) = \frac{R}{\delta}\left[log\ \left\{1+\delta\left(\frac{T}{n}-t_1\right)\right\}\right] \quad \ldots (16)$$

The maximum ordered quantity Q, is –

$$Q = I_b(0) + S$$

$$Q = R\left(t_1 + \frac{at_1^2}{2} + \frac{bt_1^3}{6}\right) + \frac{R}{\delta}log\ \left\{1+\delta\left(\frac{T}{n}-t_1\right)\right\} \quad \ldots (17)$$

Now vender's total cost is the sum of holding cost, deterioration cost, ordering cost, production cost.

V.C = Holding cost + Deterioration cost + Ordering cost + Production cost
+ Set up cost … (18)

Set up cost per cycle = S_v … (19)

The Production cost per cycle –

$$PC_v = pKDT_1 \quad \ldots (20)$$

The deterioration cost per cycle –

DC_v = (Total production – Total demand) C_v

$$DC_v = D\left(KT_1 - \left(T_1 + T_2\right)\right)C_v \quad \ldots (21)$$

The inventory holding cost per cycle –

$$HC_v = \int_0^{T_1}\left(h_1 + \alpha t\right)I_{v_1}(t)\,dt + \int_0^{T_2}\left(h_1 + \alpha t\right)I_{v_2}(t)\,dt \quad \ldots (22)$$

Let $HC_v = H_1 + H_2$ … (23)

$$H_1 = \int_0^{T_1}\left(h_1 + \alpha t\right)I_{v_1}(t)\,dt$$

$$H_1 = \int_0^{T_1}\left(h_1 + \alpha t\right)D(K-1)\left(t + \frac{at^2}{2} + \frac{bt^3}{6}\right)e^{\left(-at-\frac{bt^2}{2}\right)}dt$$

$$H_1 = D(K-1)\left\{h_1\left(\frac{T_1^2}{2} - \frac{aT_1^3}{6} - \frac{bT_1^4}{12}\right) + \alpha\left(\frac{T_1^3}{3} - \frac{aT_1^4}{8} - \frac{bT_1^5}{15}\right)\right\} \quad \ldots (24)$$

$$H_2 = D\left[h_1\left\{\frac{T_2^2}{2} + \frac{aT_2^3}{3} + \frac{bT_2^4}{8} - \frac{aT_2^3}{6} - \frac{bT_2^4}{24}\right\} + \alpha\left\{\frac{T_2^3}{6} + \frac{aT_2^4}{8} + \frac{bT_2^5}{20} - \frac{aT_2^4}{12} - \frac{bT_2^5}{40}\right\}\right] \ldots (25)$$

Put these value of H_1 and H_2 in equation (23) –

$$\begin{aligned} HC_v = {} & D(K-1)\left[\left\{h_1\left(\frac{T_1^2}{2} - \frac{aT_1^3}{6} - \frac{bT_1^4}{12}\right) + \alpha\left(\frac{T_1^3}{3} - \frac{aT_1^4}{8} - \frac{bT_1^5}{15}\right)\right\}\right] \\ & + D\left[h_1\left\{\frac{T_2^2}{2} + \frac{aT_2^3}{6} + \frac{bT_2^4}{12}\right\} + \alpha\left\{\frac{T_2^3}{6} + \frac{aT_2^4}{24} + \frac{bT_2^5}{40}\right\}\right] \end{aligned} \quad \ldots (26)$$

So the total cost function for the vender is given by –

$$\begin{aligned} V.T.C = D\Bigg[& (K-1)\left\{h_1\left(\frac{T_1^2}{2} - \frac{aT_1^3}{6} - \frac{bT_1^4}{12}\right) + \alpha\left(\frac{T_1^3}{3} - \frac{aT_1^4}{8} - \frac{bT_1^5}{15}\right)\right\} + h_1\left\{\frac{T_2^2}{2} + \frac{aT_2^3}{6} + \frac{bT_2^4}{12}\right\} \\ & + \alpha\left\{\frac{T_2^3}{6} + \frac{aT_2^4}{24} + \frac{bT_2^5}{40}\right\}\Bigg] + DC_v\left(KT_1 - \left(T_1 + T_2\right)\right) + pKDT_1 + S_v \end{aligned} \quad \ldots (27)$$

Buyer's Total cost

The Ordering cost per cycle = S_b … (28)

The Inventory holding cost per cycle

$$HC_b = R\left[h_2\left\{ \frac{t_1^2}{2} + \frac{at_1^3}{6} + \frac{bt_1^4}{12} \right\} + \alpha\left\{ \frac{t_1^3}{6} + \frac{at_1^4}{8} + \frac{bt_1^5}{20} - \frac{at_1^3}{6} - \frac{bt_1^4}{24} \right\} \right] \quad \ldots (29)$$

The Deterioration cost per cycle –

Deterioration cost = C_b (Maximum ordered quantity – Total demand)

$$DC_b = C_b\left[R\left(\frac{at_1^2}{2} + \frac{bt_1^3}{6} \right) + \frac{R}{\delta} log\left(1+\delta\left(\frac{T}{n} - t_1 \right) \right) \right] \quad \ldots (30)$$

The Shortage cost per cycle –

$$SC_b = K_b\left[\frac{R}{\delta}\left\{ \frac{-T}{n} log\left(1+\delta\left(\frac{T}{n} - t_1 \right) \right) - \left(t_1 - \frac{T}{n} \right) log\left(1+\delta\left(\frac{T}{n} - t_1 \right) \right) - \delta\left(\frac{\frac{T}{n} - t_1}{\delta} - \frac{log\left(1+\delta\left(\frac{T}{n} - t_1 \right) \right)}{\delta^2} \right) - t_1 log\left(1+\delta\left(\frac{T}{n} - t_1 \right) \right) \right\} \right]$$

$$\ldots (31)$$

The Opportunity cost per cycle

$$OC_b = O_b \frac{R}{\delta}\left[log\, n - log\left(n - nt_1\delta + T\delta \right) \right] \quad \ldots (32)$$

The Purchase cost per cycle –

$$PC_b = p_v Q$$

$$PC_b = p_v\left[R\left(t_1 + \frac{at_1^2}{2} + \frac{bt_1^3}{6} \right) + \frac{R}{\delta} log\left(1+\delta\left(\frac{T}{n} - t_1 \right) \right) \right] \quad \ldots (33)$$

Therefore the buyer's total cost is the sum of equation (29), (30), (31), (32) and (33)

B.C = Holding cost + Deterioration cost + Purchase cost + Ordering cost

+ Shortage cost + Opportunity cost

$$\begin{aligned} B.C = {} & R\left\{ h_2\left(\frac{t_1^2}{2} + \frac{at_1^3}{6} + \frac{bt_1^4}{12} \right) + \alpha\left(\frac{t_1^3}{6} + \frac{at_1^4}{8} + \frac{bt_1^5}{20} - \frac{at_1^3}{6} - \frac{bt_1^4}{24} \right) \right\} + C_b\left\{ R\left(\frac{at_1^2}{2} + \frac{bt_1^3}{6} \right) + \frac{R}{\delta} log\left(1+\delta\left(\frac{T}{n} - t_1 \right) \right) \right\} \\ & + K_b\left\{ \frac{R}{\delta}\left(\frac{-T}{n} log\left(1+\delta\left(\frac{T}{n} - t_1 \right) \right) - \left(t_1 - \frac{T}{n} \right) log\left(1+\delta\left(\frac{T}{n} - t_1 \right) \right) - \delta\left(\frac{\frac{T}{n} - t_1}{\delta} - log\frac{\left(1+\delta\left(\frac{T}{n} - t_1 \right) \right)}{\delta^2} \right) - t_1 log\left(1+\delta\left(\frac{T}{n} - t_1 \right) \right) \right) \right\} \\ & + p_v\left\{ R\left(t_1 + \frac{at_1^2}{2} + \frac{bt_1^3}{6} \right) + \frac{R}{\delta} log\left(1+\delta\left(\frac{T}{n} - t_1 \right) \right) \right\} + S_b + O_b \frac{R}{\delta}\left\{ log\, n - log\left(n - nt_1\delta + T\delta \right) \right\} \end{aligned}$$

$$\ldots (34)$$

The Integrated joint total cost function TC for the vender and the buyer is the sum of VC and BC.

$$T.C = V.C + B.C \quad \ldots (35)$$

$$\begin{aligned} T.C = {} & (c - dP_v)\left\{ (K-1)\left\{ h_1\left(\frac{T_1^2}{2} - \frac{aT_1^3}{6} - \frac{bT_1^4}{12} \right) + \alpha\left(\frac{T_1^3}{3} - \frac{aT_1^4}{8} - \frac{bT_1^5}{15} \right) \right\} + \left\{ h_1\left(\frac{T_2^2}{2} - \frac{aT_2^3}{6} - \frac{bT_2^4}{12} \right) + \alpha\left(\frac{T_2^3}{6} - \frac{aT_2^4}{24} - \frac{bT_2^5}{40} \right) \right\} \right\} \\ & + (c - dP_v)C_v\left(KT_1 - (T_1 + T_2) \right) + pKDT_1 + S_v + (c - dP_b)\left\{ h_2\left(\frac{t_1^2}{2} - \frac{at_1^3}{6} - \frac{bt_1^4}{12} \right) + \alpha\left(\frac{t_1^3}{6} + \frac{at_1^4}{8} + \frac{bt_1^5}{20} - \frac{at_1^3}{6} - \frac{bt_1^4}{24} \right) \right\} \\ & + C_b\left\{ (c - dP_b)\left(\frac{at_1^2}{2} - \frac{bt_1^3}{6} \right) + \frac{(c - dP_b)}{\delta} log\left(1+\delta\left(\frac{T}{n} - t_1 \right) \right) \right\} + K_b\left\{ \frac{(c - dP_b)}{\delta}\left(\frac{-T}{n} log\left(1+\delta\left(\frac{T}{n} - t_1 \right) \right) - \left(t_1 - \frac{T}{n} \right) log\left(1+\delta\left(\frac{T}{n} - t_1 \right) \right) \right) \right\} \\ & + p_v\left\{ (c - dP_b)\left(t_1 + \frac{at_1^2}{2} + \frac{bt_1^3}{6} \right) + \frac{(c - dP_b)}{\delta} log\left(1+\delta\left(\frac{T}{n} - t_1 \right) \right) \right\} + S_b + O_b \frac{(c - dP_b)}{\delta}\left\{ log\, n - log\left(n - nt_1\delta + T\delta \right) \right\} \end{aligned}$$

$$\ldots (36)$$

Now defuzzified the model by using signed distance method.

Fuzzy Mathematical model

Practically, demand rate and deterioration rate are imprecise. So we take c, d and a as a fuzzy number i.e., as $\hat{c}, \hat{d}, \hat{a}$. Then due to this –

$$\hat{c} = (c-\Delta_1, c, c+\Delta_2)$$
$$Where\, 0 < \Delta_1 < c\, and\, \Delta_1\Delta_2 > 0$$
$$\hat{d} = (d-\Delta_3, d, d+\Delta_4)$$
$$Where\, 0 < \Delta_3 < d\, and\, \Delta_3\Delta_4 > 0$$
$$\hat{a} = (a-\Delta_5, a, a+\Delta_6)$$
$$Where\, 0 < \Delta_5 < a\, and\, \Delta_5\Delta_6 > 0$$

And the signed distance of $\hat{c}$ is given by the relation –
$$d\left(\hat{c}, \hat{0}\right) = c + \frac{1}{4}(\Delta_2 - \Delta_1)$$
$$d\left(\hat{c}, \hat{0}\right) > 0\, and\, d\left(\hat{c}, \hat{0}\right) \in [c-\Delta_1, c+\Delta_2]$$

$d\left(\hat{c}, \hat{0}\right)$ is the estimated initial demand rate during the planning period $[0, T]$ based on the signed distance.

And the signed distance of $\hat{d}$ is given by the relation –
$$d\left(\hat{d}, \hat{0}\right) = d + \frac{1}{4}(\Delta_4 - \Delta_3)$$
$$d\left(\hat{d}, \hat{0}\right) > 0\, and\, d\left(\hat{d}, \hat{0}\right) \in [d-\Delta_3, d+\Delta_4]$$

$d\left(\hat{d}, \hat{0}\right)$ is the estimated demand rate during the planning period $[0, T]$ based on the signed distance.

And the signed distance of $\hat{a}$ is given by the relation –
$$d\left(\hat{a}, \hat{0}\right) = a + \frac{1}{4}(\Delta_6 - \Delta_5)$$
$$d\left(\hat{a}, \hat{0}\right) > 0\, and\, d\left(\hat{a}, \hat{0}\right) \in [a-\Delta_5, a+\Delta_6]$$

$d\left(\hat{a}, \hat{0}\right)$ is the estimated deterioration cost during the planning period $[0, T]$ based on the signed distance.

$$d\left[FTC\left(\hat{c}, \hat{d}, \hat{a}\right), 0\right] = F_2 + \frac{1}{4}[F_3 - F_1] \qquad \ldots (37)$$

Where $FTC\left(\hat{c}, \hat{d}, \hat{a}\right)$ the estimated fuzzified total cost and F_1, F_2, F_3 are obtained –

$$F_1=\big((c-\Delta_1)-(d-\Delta_3)P_v\big)\Big\{(K-1)\Big\{h_1\Big(\frac{T_1^2}{2}-(a-\Delta_5)\frac{T_1^3}{6}-\frac{bT_1^4}{12}\Big)+\alpha\Big(\frac{T_1^3}{3}-(a-\Delta_5)\frac{T_1^4}{8}-\frac{bT_1^5}{15}\Big)\Big\}\Big\}$$

$$+h_1\Big(\frac{T_2^2}{2}-(a-\Delta_5)\frac{T_2^3}{6}-\frac{bT_2^4}{12}\Big)+\alpha\Big(\frac{T_2^2}{6}-(a-\Delta_5)\frac{T_2^3}{24}-\frac{bT_2^4}{40}\Big)\Big\}\Big\}+\big((c-\Delta_1)-(d-\Delta_3)P_v\big)C_v\big(KT_1$$

$$-(T_1+T_2)\big)+pKDT_1+S_v+\big((c-\Delta_1)-(d-\Delta_3)P_b\big)\{h_2\Big(\frac{t_1^2}{2}-(a-\Delta_5)\frac{t_1^3}{6}-\frac{bt_1^4}{12}\Big)+\alpha\Big(\frac{t_1^3}{6}-(a-\Delta_5)\frac{t_1^4}{8}$$

$$+\frac{bt_1^5}{20}-(a-\Delta_5)\frac{t_1^3}{6}-\frac{bt_1^4}{24}\Big)\Big\}\Big\}+C_b\Big\{\big((c-\Delta_1)-(d-\Delta_3)P_b\big)\Big((a-\Delta_5)\frac{t_1^2}{2}-\frac{bt_1^3}{6}\Big)+\frac{\big((c-\Delta_1)-(d-\Delta_3)P_b\big)}{\delta}$$

$$\log\Big(1+\delta\Big(\frac{T}{n}-t_1\Big)\Big)\Big\}+K_b\Big\{\frac{\big((c-\Delta_1)-(d-\Delta_3)P_b\big)}{\delta}\Big(\frac{-T}{n}\log\Big(1+\delta\Big(\frac{T}{n}-t_1\Big)\Big)-\Big(t_1-\frac{T}{n}\Big)$$

$$\log\Big(1+\delta\Big(\frac{T}{n}-t_1\Big)\Big)\Big)\Big\}+p_v\Big\{\big((c-\Delta_1)-(d-\Delta_3)P_b\big)\Big(t_1-(a-\Delta_5)\frac{t_1^2}{2}-\frac{bt_1^3}{6}\Big)+\frac{\big((c-\Delta_1)-(d-\Delta_3)P_b\big)}{\delta}$$

$$\log\Big(1+\delta\Big(\frac{T}{n}-t_1\Big)\Big)\Big\}+S_b+O_b\frac{\big((c-\Delta_1)-(d-\Delta_3)P_b\big)}{\delta}\{\log n-\log(n-nt_1\delta+T\delta)\} \quad \ldots (38)$$

$$F_2=(c-dP_v)\Big\{(K-1)\Big\{h_1\Big(\frac{T_1^2}{2}-\frac{aT_1^3}{6}-\frac{bT_1^4}{12}\Big)+\alpha\Big(\frac{T_1^3}{3}-\frac{aT_1^4}{8}-\frac{bT_1^5}{15}\Big)\Big\}+h_1\Big(\frac{T_2^2}{2}-\frac{aT_2^3}{6}-\frac{bT_2^4}{12}\Big)$$

$$+\alpha\Big(\frac{T_2^2}{6}-\frac{aT_2^3}{24}-\frac{bT_2^4}{40}\Big)\Big\}\Big\}+(c-dP_v)C_v\big(KT_1-(T_1+T_2)\big)+pKDT_1+S_v+(c-dP_b)$$

$$\{h_2\Big(\frac{t_1^2}{2}-\frac{at_1^3}{6}-\frac{bt_1^4}{12}\Big)+\alpha\Big(\frac{t_1^3}{6}-\frac{at_1^4}{8}+\frac{bt_1^5}{20}-\frac{at_1^3}{6}-\frac{bt_1^4}{24}\Big)\Big\}+C_b\{(c-dP_b)\Big(\frac{at_1^2}{2}-\frac{bt_1^3}{6}\Big)+\frac{(c-dP_b)}{\delta}$$

$$\log\Big(1+\delta\Big(\frac{T}{n}-t_1\Big)\Big)\Big\}+K_b\Big\{\frac{(c-dP_b)}{\delta}\Big(\frac{-T}{n}\log\Big(1+\delta\Big(\frac{T}{n}-t_1\Big)\Big)-\Big(t_1-\frac{T}{n}\Big)\log\Big(1+\delta\Big(\frac{T}{n}-t_1\Big)\Big)\Big)\Big\}$$

$$+p_v\{(c-dP_b)\Big(t_1-\frac{at_1^2}{2}-\frac{bt_1^3}{6}\Big)+\frac{(c-dP_b)}{\delta}\log\Big(1+\delta\Big(\frac{T}{n}-t_1\Big)\Big)\Big\}+S_b+O_b\frac{(c-dP_b)}{\delta}\{\log n-\log(n-nt_1\delta+T\delta)\} \quad \ldots (39)$$

$$F_3=\big((c+\Delta_2)-(d+\Delta_4)P_v\big)\Big\{(K-1)\Big\{h_1\Big(\frac{T_1^2}{2}-(a+\Delta_6)\frac{T_1^3}{6}-\frac{bT_1^4}{12}\Big)+\alpha\Big(\frac{T_1^3}{3}-(a+\Delta_6)\frac{T_1^4}{8}-\frac{bT_1^5}{15}\Big)\Big\}\Big\}$$

$$+h_1\Big(\frac{T_2^2}{2}-(a+\Delta_6)\frac{T_2^3}{6}-\frac{bT_2^4}{12}\Big)+\alpha\Big(\frac{T_2^2}{6}-(a+\Delta_6)\frac{T_2^3}{24}-\frac{bT_2^4}{40}\Big)\Big\}\Big\}+\big((c+\Delta_2)-(d+\Delta_4)P_v\big)C_v\big(KT_1$$

$$-(T_1+T_2)\big)+pKDT_1+S_v+\big((c+\Delta_2)-(d+\Delta_4)P_b\big)\{h_2\Big(\frac{t_1^2}{2}-(a+\Delta_6)\frac{t_1^3}{6}-\frac{bt_1^4}{12}\Big)+\alpha\Big(\frac{t_1^3}{6}-(a+\Delta_6)\frac{t_1^4}{8}$$

$$+\frac{bt_1^5}{20}-(a+\Delta_6)\frac{t_1^3}{6}-\frac{bt_1^4}{24}\Big)\Big\}\Big\}+C_b\Big\{\big((c+\Delta_2)-(d+\Delta_4)P_b\big)\Big((a+\Delta_6)\frac{t_1^2}{2}-\frac{bt_1^3}{6}\Big)+\frac{\big((c+\Delta_2)-(d+\Delta_4)P_b\big)}{\delta}$$

$$\log\Big(1+\delta\Big(\frac{T}{n}-t_1\Big)\Big)\Big\}+K_b\Big\{\frac{\big((c+\Delta_2)-(d+\Delta_4)P_b\big)}{\delta}\Big(\frac{-T}{n}\log\Big(1+\delta\Big(\frac{T}{n}-t_1\Big)\Big)-\Big(t_1-\frac{T}{n}\Big)$$

$$\log\Big(1+\delta\Big(\frac{T}{n}-t_1\Big)\Big)\Big)\Big\}+p_v\Big\{\big((c+\Delta_2)-(d+\Delta_4)P_b\big)\Big(t_1-(a+\Delta_6)\frac{t_1^2}{2}-\frac{bt_1^3}{6}\Big)+\frac{\big((c+\Delta_2)-(d+\Delta_4)P_b\big)}{\delta}$$

$$\log\Big(1+\delta\Big(\frac{T}{n}-t_1\Big)\Big)\Big\}+S_b+O_b\frac{\big((c+\Delta_2)-(d+\Delta_4)P_b\big)}{\delta}\{\log n-\log(n-nt_1\delta+T\delta)\} \quad \ldots (40)$$

Now we will discuss the numerical example for crisp and fuzzified model

Numerical Example

$a=.01,\quad b=.002,\quad c=1500,\quad d=8,\quad F_v=0.25,\quad F_b=0.25$

$S_v=400,\quad S_b=60,\quad p_v=25,\quad p_b=40,\quad \delta=0.015,\quad K_b=18$

$O_b=250,\quad T=40$

Total cost for different value of n

n	k	T_2	t_1	T_1	T.C.
1	2	14.8081	20.1982	25.9341	8219.31
2	2	17.8346	24.7518	24.1757	11514.43
3	2	19.2012	25.1327	22.8415	13819.81
4	2	22.1342	27.9854	18.1136	12040.704
5	2	25.4257	30.1146	16.7525	10181.89
6	2	28.1137	30.8172	10.1342	9872.02
7	2	32.1345	31.2740	9.9182	9185.81

Conclusion

When the environment is fuzzy, there are many modifications in the problem which was previously defined in a crisp sense. Sometimes it so happens, that the decision maker does not even want to maximize or minimize any objective function, rather he might want to achieve some aspiration levels which might not be even definable crisply. In such cases, fuzzy formulation of the problem comes in handy for the decision maker. The dependence of the sale of any item on its selling price is not a new concept, but a common sense conclusion. It is a general observation that an increase in the selling price of the commodity will deter its customer's from opting that item in future. In this paper we developed an inventory model in which there is no replacement or repair of deteriorated units. Inventory is continuously reviewed. Demand rate is taken as price sensitive. In this study shortages are allowed for buyers only, which is partially backlogged. In this model we consider single vendor and a single buyer for a single product. The production rate is finite and is greater than the sum of all the buyer demand. The deterioration rate is taken as linear time dependent. The model has been explored analytically and numerically.

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